Efficient Analysis of Waveguide Components by FDTD Combined with Time Domain Modal Expansion

F. Alimenti, P. Mezzanotte, *Member, IEEE,* L. Roselli, *Member, IEEE,* and R. Sorrentino, *Fellow, IEEE*

***Abstract*—A novel Finite Difference Time Domain (FDTD) scheme is proposed for the analysis of waveguide components. The method consists of combining the conventional 3D with the 1D-FDTD algorithm resulting from the time domain modal expansion in uniform waveguides. The new algorithm has been validated on a simple test example showing that the same accuracy can be obtained with a substantial improvement in the numerical efficiency.**

I. Introduction

The FDTD [1], [2] is a well-established numerical technique for the analysis of microwave structures. The recent development of modal absorbing boundary conditions (ABC's) [3], [4] has extended the applicability of the method to waveguide circuits by eliminating errors due to the truncation of the computational domain. Nonetheless, practical problems involve considerable or even unaffordable numerical efforts, because of the large extension of the computational domain and thus the large number of cells and unknowns involved by the discretization of a 3D structure. A graded mesh and a subgriding can only alleviate this difficulty.

It can be observed, however, that waveguide components generally consist of a small number of discontinuities and/or junctions (possibly with arbitrary geometries), connected by uniform waveguide lengths. The latter have usually simple (e.g., rectangular) cross-sections with analytically known modal spectra. Using a time domain modal expansion of the electromagnetic (EM) field propagating in the waveguides, a novel FDTD scheme is proposed in this letter. The conventional 3D formulation is retained for the discontinuity regions, while a 1D formulation is employed for the waveguides. In this manner, a substantial reduction of the mesh size, thus of the number of unknowns, is achieved leading to a significant improvement of the numerical efficiency, in terms of both CPU time and memory storage requirements.

II. Formulation of the Method

The method is illustrated at the example of the simple structure of Fig. 1, shown at the top of the next page, where two arbitrary discontinuities (regions 1 and 3) are connected by a uniform waveguide (region 2).

Manuscript received March 29, 1995.

The authors are with the Institute of Electronics, University of Perugia, S. Lucia Canetola I-06131, Perugia, Italy.

IEEE Log Number 9414134.

Let us first consider the uniform waveguide region comprised between the reference planes at $z = k_1$ and $z = k_2$. Rather than applying the conventional FDTD scheme, we can use modal analysis.

Modal expansion of the EM field in a waveguide is usually performed in frequency domain, but a hollow waveguide can also be applied in time domain since the eigenvectors are independent of frequency. For a z-directed waveguide, the transverse components of the EM field can be expressed as

$$\vec{E}_\tau(x, y, z, t) = \sum_n V_n(z, t)\vec{e}_n(x, y) \tag{1}$$

$$\vec{H}_\tau(x, y, z, t) = \sum_n I_n(z, t)\vec{h}_n(x, y) \tag{2}$$

where $\vec{e}_n$ and $\vec{h}_n$ are orthonormalized modal eigenvectors.

The EM field amplitudes (equivalent voltages V_n and currents I_n) are related to the field distribution by

$$V_n(z, t) = \iint_S \vec{E}_\tau(x, y, z, t) \cdot \vec{e}_n(x, y)\, dx\, dy \tag{3}$$

$$I_n(z, t) = \iint_S \vec{h}_\tau(x, y, z, t) \cdot \vec{h}_n(x, y)\, dx\, dy, \tag{4}$$

and satisfy the following differential equation:

$$\frac{\partial^2 f_n}{\partial z^2} - \frac{1}{c_0^2}\frac{\partial^2 f_n}{\partial t^2} - k_{cn}^2 f_n = 0 \tag{5}$$

in (3) and (4) S is the waveguide cross-section; in (5) f_n stands for either V_n or I_n, k_{cn}^2 is the eigenvalue of the nth mode, and c_0 is the wave velocity in a vacuum. f_n represents the time varying amplitude of the nth mode at the z-coordinate. As such, it may contain a range of frequencies partly above and partly below cutoff. Equation (5) can be discretized in space (z) and time (t) according to the central difference scheme

$$f_{n,k}^{t+1} = \frac{c_0^2 \Delta t^2}{\Delta z^2}\left(f_{n,k+1}^t - 2f_{n,k}^t + f_{n,k-1}^t\right) - c_0^2 \Delta t^2 k_{cn}^2 f_{n,k}^t + 2f_{n,k}^t - f_{n,k}^{t-1}. \tag{6}$$

Equation (6) governs the time-discrete evolution of the EM field of the nth mode in the waveguide region. The knowledge of the modal eigenvectors $\vec{e}_n$, $\vec{h}_n$ has reduced the analysis to a very simple (and very fast) one-dimensional FDTD algorithm for each mode.

1051-8207/95$04.00 © 1995 IEEE

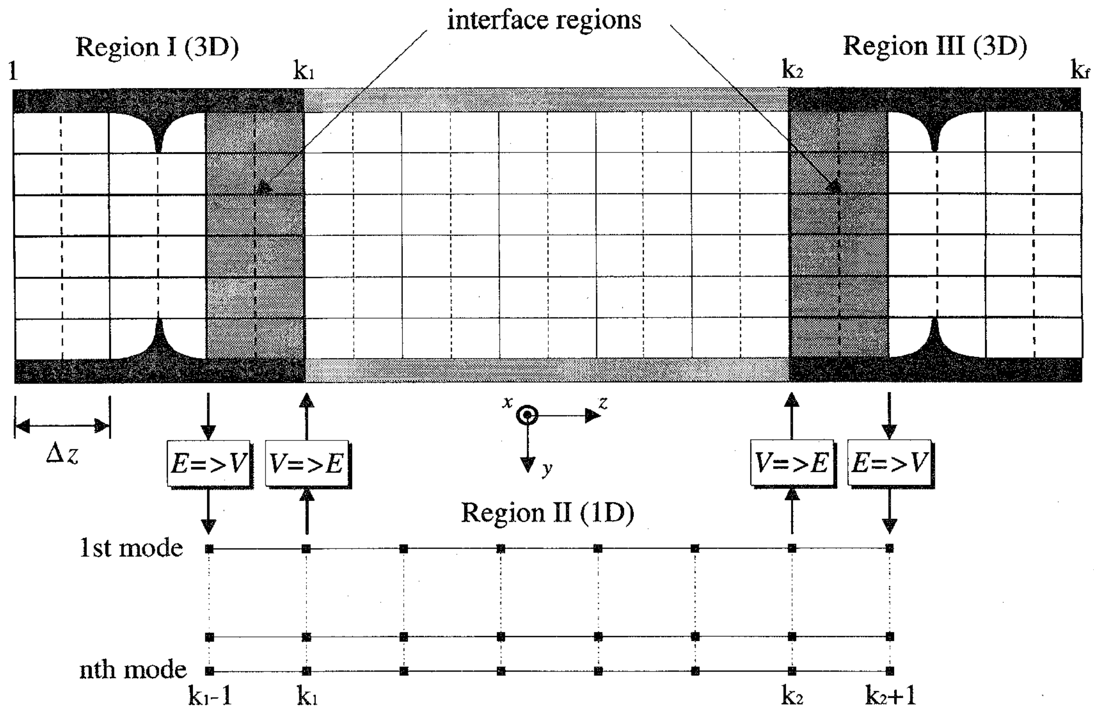

Fig. 1. Interfacing 1D with 3D grids.

TABLE I
CONVENTIONAL FDTD TIME MARCHING SOLUTION

1. For t from 0 to final time step
1.1. Updating of the H-field components
1.2. Application of the H-field boundary conditions
1.3. Updating of the E-field components
1.4. Application of the E-field boundary conditions

TABLE II
MODIFIED FDTD TIME MARCHING SOLUTION

1. For t from 0 to final time step	
1.1. For k = 1 ÷ (k₁-1/2) and k = (k₂+1/2) ÷ (k_f-1/2)	
1.1.1. Updating the H-field components	*(3D)*
1.2. Application of the H boundary conditions	
1.3. For k = 2 ÷ (k₁-1) and k = (k₂+1) ÷ (k_f-1)	
1.3.1. Updating of the E-field components	*(3D)*
1.4. For k = k₁ ÷ k₂	
1.4.1. Updating of the voltages	*(1D)*
1.5. V(k₁) → E(k₁), V(k₂) → E(k₂)	*Eq. (1)*
1.6. E(k₁-1) → V(k₁-1), E(k₂+1) → V(k₂+1)	*Eq. (3)*
1.7. Application of the E boundary conditions	

In practical cases, a limited number of modes need to be included in the waveguide region, depending on 1) the frequency band of the source, and 2) the proximity of the reference planes to the discontinuities. Shifting such planes closer to the discontinuities in order to reduce the 3D domain requires additional modes to be included in the 1D simulation of the waveguide region. Determining the best compromise to minimize the computational effort, however, is outside the scope of this letter.

The conventional 3D-FDTD scheme is used for the discontinuity regions. As shown in Fig. 1, they correspond to the spatial index k being comprised in the intervals 1 to k_1 (region 1), and k_2 to k_f (region 3), while the waveguide region corresponds to $k = k_1$ to k_2. Equations (1)–(4) can be used to interface the 3D with the 1D meshes. Specifically, (1) and (2) allow the field distributions at the reference planes of the discontinuities to be computed from the voltage and current amplitudes at the waveguide ends, while (3) and (4) convert the field distributions at the reference planes into the voltage and current amplitudes at the waveguide ends.

To illustrate the practical implementation of the mixed 1D–3D algorithm, consider first the conventional time marching solution of the FDTD leapfrog algorithm. As illustrated in Table I, the H- and E-fields are updated alternately in time, and the respective boundary conditions on the H- or E-field, are applied at proper time instants. The 1D–3D time marching scheme is described in Table II. It differs from the conventional one for the spatial loop $k = k_1$ to k_2 (Step 1.4) and for Steps 1.5 and 1.6 that correspond to interlacing the 3D with 1D scheme. Observe that the 3D and 1D subdomains partially overlap at $k = (k_1 - 1)$ to k_1 [and $k = k_2$ to $(k_2 + 1)$]. This is necessary for updating the E-field in k_1 (and k_2) using (1) and for computing the voltage in $k_2 - 1$ (and $k_2 + 1$) using (3).

Note that only the voltage amplitude has been used in the waveguide region. Alternatively, only the current amplitude could also be used.

It could be observed that rather than by a 1D FD scheme, the uniform waveguide could also be described analytically either in time [5] or frequency domains (e.g., scattering matrix). In both cases, however, a considerable additional effort would be required for the computation of convolution

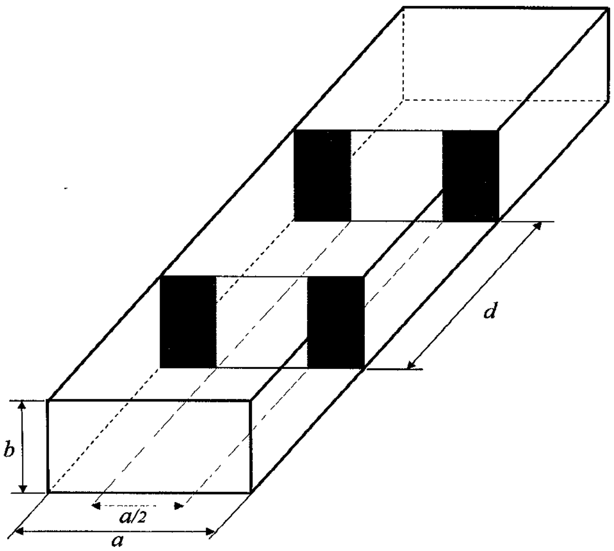

Fig. 2. The simulated structure.

integrals and/or time-to-frequency transforms. Observe also that the computation of a terminal description such as the generalized scattering matrix requires a number of simulations equal to the matrix size, and the implementation of multimodal ABC's.

III. Results

A simple structure consisting of two inductive irises in a WR90 rectangular waveguide has been assumed in order to validate the method (see Fig. 2). The reference planes and the source frequency range have been chosen in such a way that only the dominant TE_{10} mode need to be considered. A simple transmission line is thus sufficient to model the EM field propagation in the waveguide.

The distance d between the two irises has been chosen to be $3\lambda/2$ at about 12.5 GHz. The structure has been analyzed using both the conventional and the new 3D–1D scheme. In the former case a uniform mesh with $11 * 41 * 87$ cells has been used, the spatial and time steps being $\Delta x = 1.016$ mm, $\Delta y = 0.5715$ mm, $\Delta z = 1$ mm, and $\Delta t = 1$ ps. With the new formulation, the waveguide region has been simulated assuming unimodal propagation.

The waveguide has been terminated by two unimodal absorbing walls [4], [5] 15 mm away from the irises to avoid higher-order modes interaction and excited by a carrier signal of 10.3 GHz modulated by a Gaussian pulse 4.2 GHz wide at the 5% of the maximum power.

The results computed by both approaches are plotted in Fig. 3. The corresponding curves are perfectly overlapped,

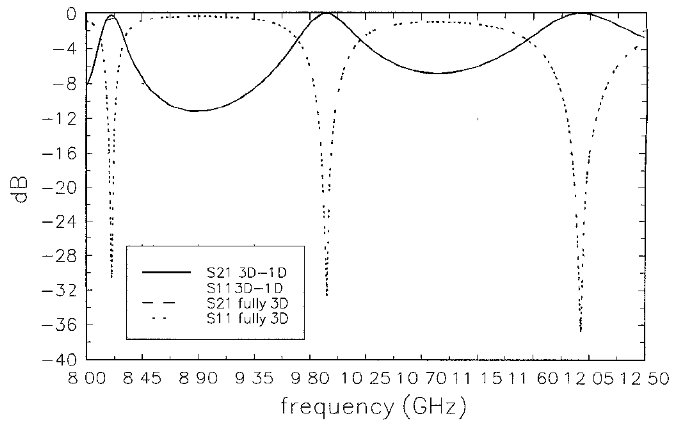

Fig. 3. Comparison between 3D and 1D-3D FDTD simulations of the structure in Fig. 2.

but the waveguide region has been discretized by just 33 unknowns instead of 89 298 as required by the conventional FDTD, with corresponding time and memory savings.

IV. Conclusion

A novel computational scheme that combines the conventional FDTD algorithm with the modal expansion of the EM field in a waveguide has been proposed for the efficient analysis of waveguide components. The modal expansion allows uniform waveguide lengths to be modeled as transmission lines in time domain, thus reducing from 3D to 1D the complexity of the problem. The mixed 1D-3D scheme has been validated at the example of a simple rectangular waveguide problem, where unimodal propagation has been assumed in the uniform waveguide region. The inclusion of higher-order modes as well as the generalization to arbitrary waveguide cross sections can be made in a straightforward manner.

References

[1] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 302–307, May 1966.

[2] A. Taflove and M. E. Brodwin, "Numerical solution of steady-state electromagnetic scattering problems using the time-dependent Maxwell's equations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 623–630, Aug. 1975.

[3] F. Moglie, T. Rozzi, P. Marcozzi, and A. Schiavoni, "A new termination condition for the application of FDTD technique to discontinuity problems in close homogeneous waveguide," *IEEE Microwave and Guided Wave Lett.*, vol. 2, no. 12, pp. 475–477, Dec. 1992.

[4] Volker Grimm, *Absorbing Boundary Conditions for the FDTD-Method in a Hollow Waveguide.* Perugia, Italy: Studienarbeit, Apr. 4, 1994.

[5] M. Righi, M. Mongiardo, R. Sorrentino, and W. Hoefer, "Efficient TLM diakoptics for separable structures," in *Proc. Int. MTT-S*, Atlanta, GA, June 14–18, 1993, vol. 1, pp. 425–428.